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Symmetries and Generalisations of Tri-Bimaximal Neutrino Mixing.

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Abstract

Tri-bimaximal mixing is a specific lepton mixing ansatz, which has been shown to account very successfully for the established neutrino oscillation data. Working in a particular basis (the 'circulant basis'), we identify three independent symmetries of tri-bimaximal mixing, which we exploit to set the tri-bimaximal hypothesis in context, alongside some simple, phenomenologically interesting CP-conserving and CP-violating generalisations.

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1. Introduction

Tri-bimaximal mixing [1] is a very successful lepton mixing ansatz, which has already attracted a degree of attention in the literature [2]. In the standard parametrisation [3], tri-bimaximal mixing may be specified by: $\theta_{12} = \sin^{-1}(1/\sqrt{3})$, $\theta_{23} = -\pi/4$ and $\theta_{13} = 0$, with no CP-violating phase. Tri-bimaximal mixing builds squarely on all of the most promising phenomenological ideas which have preceded it [4] and readily accounts for all of the best-established neutrino oscillation results to date [5] [6] [7].

Despite these sucesses we have no reason to suppose that tri-bimaximal mixing will prove to be exactly right in every detail, and we seek therefore to generalise the original tri-bimaximal hypothesis, so as to parametrise possible deviations in simple and meaningful ways, which we hope will be useful in developing experimental tests. We begin by reviewing the symmetries inherent in the tri-bimaximal scheme, which will lead us to identify generic features which will form the basis of our generalisations.

2. Tri-Bimaximal Mixing and the Circulant Basis

Symmeties are usually thought to be best studied at the level of the mass-matrices, which are naturally referred to a 'weak' basis (ie. a basis which leaves the charged-current weak-interaction diagonal and universal). Furthermore, by restricting consideration to left-handed fields only, we may take our mass-matrices (squared) to be hermitian. Following Ref. [1], we will work in a particular weak basis in which the mass-matrix for the charged leptons takes the familiar 3×3 'circulant' form [8]:

$$M_l^2 = \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix} \tag{1}$$

where the constants a, b and b^* encode the lepton masses as follows:

$$\begin{aligned} a &= \frac{m_e^2}{3} + \frac{m_\mu^2}{3} + \frac{m_\tau^2}{3} \\ b &= \frac{m_e^2}{3} + \frac{m_\mu^2 \omega}{3} + \frac{m_\tau^2 \bar{\omega}}{3} \\ b^* &= \frac{m_e^2}{3} + \frac{m_\mu^2 \bar{\omega}}{3} + \frac{m_\tau^2 \omega}{3} \end{aligned} \tag{2}$$

($\omega = \exp(i2\pi/3)$ and $\bar{\omega} = \exp(-i2\pi/3)$ are complex cube roots of unity).

In the above basis (the 'circulant basis') the charged-lepton mass-matrix (Eq. 1) is clearly invariant under cyclic permutations of the three generation indices. Note, however, that invariance under *odd* permuations of the generation indices (ie. generation interchange) would require that *odd* permutations are performed simultaneously with a complex conjugation (see Section 3 below).

Thus far, we have simply chosen a basis, and it is the form of the neutrino mass matrix in this basis which determines the observable mixing. Since circulant matrices of identical order always commute, we cannot take the neutrino mass-matrix to be also a 3×3 circulant and obtain non-trivial mixing. We were led therefore to postulate [1] a neutrino mass-matrix which is invariant under cyclic permutations of only *two* out of the three generations, ie. a 2×2 circulant with one generation (which was taken to be generation 2 in this basis) isolated in the mass-matrix by four 'texture zeroes' [9], yielding the effective block-diagonal form:

$$M_\nu^2 = \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix}. \tag{3}$$

In Eq. 3 the constants x, y and z (y negative) encode the neutrino masses as follows:

$$\begin{aligned} x &= \frac{m_1^2}{2} + \frac{m_3^2}{2} \\ y &= \frac{m_1^2}{2} - \frac{m_3^2}{2} \\ z &= m_2^2 \end{aligned} \tag{4}$$

Note that the neutrino mass matrix Eq. 3 is real and symmetric (as well as being, by construction, a 2×2 circulant in the $1-3$ index subset). With y real, the neutrino mass-matrix Eq. 3 is invariant under the *odd* permutation corresponding to the generation interchange $1 \leftrightarrow 3$, performed with or without a complex conjugation (cf. Eq. 1 above).

The charged-lepton mass-matrix M_l^2 (Eq. 1) and the neutrino mass-matrix M_ν^2 (Eq, 3) are diagonalised by threefold maximal and twofold maximal unitary matrices U_l and U_ν, respectively [1]. The MNS matrix [10] is then given by $U_l^\dagger U_\nu = U$:

$$\begin{array}{cc} & \\ e \\ \mu \\ \tau \end{array} \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{\bar{\omega}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} \end{pmatrix} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} \sqrt{\frac{1}{2}} & 0 & \sqrt{\frac{1}{2}} \\ 0 & 1 & 0 \\ \sqrt{\frac{1}{2}} & 0 & -\sqrt{\frac{1}{2}} \end{pmatrix} \end{array} = \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\ \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\frac{i}{\sqrt{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \frac{i}{\sqrt{2}} \end{pmatrix} \end{array} \tag{5}$$

which is tri-bimaximal mixing in a particular phase convention. Note that the tri-bimaximal mixing matrix (Eq. 5 - RHS) has two rows (row 2 and row 3) which are complex conjugates of each other (so that corresponding elements are equal in modulus). This is readily traced to the fact that the matrix $U_l^\dagger$ (Eq. 5 - LHS) has likewise two rows (row 2 and row 3) complex-conjugate, while the matrix U_ν is real. It will prove useful to observe (see Section 3) that the matrix $U_l^\dagger$ (Eq. 5 - LHS) has also two *columns* (columns 1 and 3) complex conjugate.

3. CP-Conservation and Tri-Phi-Maximal Mixing

In the circulant basis (see Section 2, above) a number of generic features of tri-bimaximal mixing (Eq. 5) manifest themselves very simply in the form of the neutrino mass matrix (Eq. 3). Perhaps the most significant feature of tri-bimaximal mixing is the predicted absence of CP-violation in neutrino oscillations. If CP is conserved, the MNS matrix is orthogonal (or may be taken[1] to be orthogonal). As remarked in Section 2, the unitary matrix $U_l^\dagger$ (Eq. 5 - LHS) has two columns (columns 1 and 3) complex conjugates of each other. If the MNS matrix is orthogonal (ie. $U = O$), we have $U_\nu = U_l O$ which then gives U_ν with two *rows* (rows 1 and 3) complex conjugates of each other. Since $M_\nu^2 = U_\nu \mathrm{diag}(m_1^2,\ m_2^2,\ m_3^2) U_\nu^\dagger$ this symmetry must then be manifest in the neutrino mass-matrix so that, in particular, a sufficiently general form for the neutrino mass matrix in the circulant basis, yielding no CP-violation in neutrino oscillations, may be written[2]:

$$M_\nu^2 = \begin{pmatrix} x & w & y^* \\ w^* & z & w \\ y & w^* & x \end{pmatrix}. \tag{6}$$

The form Eq. 6 generalises Eq. 3, and also mirrors the circulant form Eq. 1 but with the 2nd generation distinguished (when $z = x$ and $y = w$ Eq. 6 becomes circulant). The six real parameters correspond to the three masses and three real mixing angles. The important point however is that Eq. 6 exhibits the symmetry of Eq. 1 under the exchange of generations $1 \leftrightarrow 3$ performed simutaneously with a complex conjugation. Indeed, in the circulant basis, it is the invariance of *all* the leptonic terms under this combined operation that ensures $J_{CP} = 0$ (since Im det$[M_l^2, M_\nu^2]$ changes sign).

Starting from Eq. 6, if we take w and y real, we immediately recover Altarelli-Feruglio mixing [12] with $\tan 2\theta = 2\sqrt{2}(x + y - z - w)/(x + y - z + 8w)$. If we consider only the combination $w^2 y$ real (so that w and y have correlated phase, with $w = |w| \exp(-i\phi)$, $y = -|y| \exp(i2\phi)$ complex) we obtain a simple two parameter

[1] The phases of the charged-lepton mass-eigenstates are entirely unphysical and may be re-defined at will. Also, the phases of the neutrino mass-eigenstates have no influence on the form of the neutrino mass-matrix (squared) M_ν^2 as defined here, ie. no influence on Eq. 6.

[2] It should perhaps be said that the similarity of Eq. 6 of the present paper to Eq. 31 of Ref. [11] appears to be somewhat accidental. We remind the reader that we are working here in the 'circulant basis' (defined in Section 2, above) and not in the lepton flavour basis as in Ref. [11].

generalisation of Altarelli-Feruglio mixing with $J_{CP} = 0$, but $|U_{e3}| \neq 0$ in general. Taking $|w| \to 0$ (as required for tri-bimaximal mixing) but keeping $y = -|y| \exp(i2\phi)$ complex with a fixed phase-angle ϕ, we obtain a block-diagonal, complex neutrino mass-matrix, generalising Eq. 3, which will lead to 'tri-ϕmaximal' mixing (below):

$$M_\nu^2 = \begin{pmatrix} x & 0 & y^* \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix}. \tag{7}$$

The neutrino masses are given similarly to Eq. 4 (but with y replaced by $-|y|$):

$$\begin{aligned} x &= \frac{m_1^2}{2} + \frac{m_3^2}{2} \\ |y| &= \frac{m_3^2}{2} - \frac{m_1^2}{2} \\ z &= m_2^2 \end{aligned} \tag{8}$$

and we obtain the simple, one-parameter, CP-conserving generalisation of tri-bimaximal mixing (depending on the phase-angle ϕ) referred to here as 'tri-ϕmaximal' mixing:

$$U \quad = \quad \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \quad\nu_1 \quad & \quad \nu_2 \quad & \quad \nu_3 \quad \end{array} \\ \begin{pmatrix} \sqrt{\frac{2}{3}} \cos\phi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \sin\phi \\ -\frac{\cos\phi}{\sqrt{6}} - \frac{\sin\phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}} \\ -\frac{\cos\phi}{\sqrt{6}} + \frac{\sin\phi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\phi}{\sqrt{2}} - \frac{\sin\phi}{\sqrt{6}} \end{pmatrix} \end{array}. \tag{9}$$

The phase angle ϕ must satisfy $|\sin\phi| \lesssim 0.2$ to fit the reactor data [7], while there is minimal impact on the fit to the atmospheric data [6]. Tri-ϕmaximal mixing has $U_{e3} \neq 0$, but retains the symmetries $|U_{e2}| = |U_{\mu 2}| = |U_{\tau 2}|$ and $J_{CP} = 0$. Note that in tri-ϕmaximal mixing the symmetry of two rows complex conjugate is sacrificed (compare Eq. 9 rows 2 and 3 with Eq. 5 - RHS, rows 2 and 3). Clearly tri-ϕmaximal mixing reduces to tri-bimaximal mixng in the limit $\phi \to 0$.

4. CP-Violation and Tri-Chi-Maximal Mixing

In the circulant basis, it will be enough to require that the neutrino mass matrix be real (ie. symmetric, since our mass matrices are hermitian) to ensure that two rows of the MNS matrix have corresponding elements which are equal in modulus [3], just as in Eq. 1 (rows 2 and 3 in this case). Such mixing matrices form an already

[3]In general, if the MNS matrix has two rows with corresponding elements equal in modulus, then by approriate re-phasings, the two rows may always be taken to be complex-congugate to each other, with the remaining row taken to be purely real.

interesting (two-parameter) generalisation of tri-bimaximal mixing, with a form of mu-tau universality. This time (cf. Section 3 above) the proof depends on the unitary matrix $U_l^\dagger$ (Eq. 5) having two *rows* (row 2 and row 3) complex-conjugate. The theorem follows immediately by noting that a real symmetric matrix may always be diagonalised by an orthogonal matrix (ie. $U_\nu = O$, with O an orthogonal matrix) so that the resulting MNS matrix $U \equiv U_l^\dagger U_\nu = U_l^\dagger O$, is necessarily also of the form Eq. 5, with rows 2 and 3 complex-conjugate and row 1 purely real. Clearly the resulting mixing matrix is invariant under interchange of row 2 and row 3 performed simultaneously with a complex-conjugation. This form of mu-tau univesality implies strict mu-tau symmetry for CP-even observables (eg. disappearance probabilities), but has CP-odd observables (eg. asymmetries in appearance probabiliites) changing sign.

Of course, non-zero CP-violation means sacrificing the symmetry (Eq. 6) of the neutrino mass-matrix under $1 \leftrightarrow 3$ interchange together with complex-conjugation. If we do this taking the neutrino mass-matrix to be real as discussed above, and again demanding effective block-diagonal form, just as for tri-bimaximal mixing, we are immediately led to the neutrino mass-matrix for 'tri-χmaximal' mixing:

$$M_\nu^2 = \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & w \end{pmatrix} \tag{10}$$

The real constants x, y, z and w now encode the neutrino masses and one mixing angle χ as follows:

$$\begin{aligned} x &= \frac{m_1^2 + m_3^2}{2} - \frac{m_3^2 - m_1^2}{2} \sin 2\chi \\ w &= \frac{m_1^2 + m_3^2}{2} + \frac{m_3^2 - m_1^2}{2} \sin 2\chi \\ z &= m_2^2 \end{aligned} \tag{11}$$

where $\cot 2\chi = 2y/(x-w)$, leading to our second simple one-parameter generalisation of tri-bimaximal mixing, called here 'tri-χmaximal' mixing:

$$U \;=\; \begin{array}{c} \\ e \\ \mu \\ \tau \end{array} \begin{array}{c} \begin{array}{ccc} \quad\nu_1 \quad & \quad\nu_2\quad & \quad\nu_3\quad \end{array} \\ \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\chi & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}\sin\chi \\ -\frac{\cos\chi}{\sqrt{6}} - i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & i\frac{\cos\chi}{\sqrt{2}} - \frac{\sin\chi}{\sqrt{6}} \\ -\frac{\cos\chi}{\sqrt{6}} + i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -i\frac{\cos\chi}{\sqrt{2}} - \frac{\sin\chi}{\sqrt{6}} \end{pmatrix} \end{array} \tag{12}$$

Again, we have $|\sin\chi| \lesssim 0.2$ in order to fit the reactor data [7]. Tri-χmaximal mixing has non-zero $U_{e3} = \sqrt{2/3}\sin\chi$ and maximal CP-violation (for fixed $|U_{e3}|$) with the Jarlskog invariant [13] given by $J_{CP} = \sin 2\chi/(6\sqrt{3})$. As expected, Eq. 12 has rows 2 and 3 complex conjugates of each other. Tri-χmaximal mixing Eq.12 and tri-ϕmaximal mixing Eq. 9 are clearly very closely related (they are identical interchanging $\chi \leftrightarrow \phi$, apart from the factors of i). Again, tri-χmaximal mixing (Eq. 12) reduces to tri-bimaximal mixing in the limit $\chi \to 0$.

Finally, we note that there is always the possibility of specialising our mixings by imposing additional constraints. An amusing specialisation of tri-χmaximal mixing would be to require $y = z - (x + w)/2$, which leads immediately to: $\sin\chi = \sqrt{\Delta m^2_{21}/\Delta m^2_{31}} \sim 0.13$, certainly consistent with current experimental limits [7], and holding out the promise of observable CP-violation in future experiments [14].

5. Discussion

We have identified three symmetries of tri-bimaximal mixing. In the circulant basis (where the charged-lepton mass-matrix takes a simple 3×3 circulant form) these symmetries may be (separately) implemented by taking the neutrino mass-matrix to be i) real, ii) invariant under $1 \leftrightarrow 3$ interchange with complex conjugation and iii) effective block-diagonal in the $1, 3$ index subset. At the level of the mixing matrix these symmetries correspond to the properties i) two rows (rows 2 and 3) complex conjugate (equal in modulus), ii) no CP-violation in neutrino oscillations ($J_{CP} = 0$) and iii) trimaximal mixing for solar neutrinos, ie. $|U_{e2}| = |U_{\mu 2}| = |U_{\tau 2}| = 1/\sqrt{3}$.

Together with the Altarelli-Feruglio hypothesis [12], tri-ϕmaximal and tri-χmaximal mixing form the complete set of natural one-parameter generalisations of tri-bimaximal mixing, defined by dropping any one (or retaining any two) of the above three symmetries. Thus tri-ϕmaximal mixing (Eq. 9) retains ii) $J_{CP} = 0$ and iii) $|U_{e2}| = |U_{\mu 2}| = |U_{\tau 2}| = 1/\sqrt{3}$, but drops i) two-rows complex-conjugate, while tri-χmaximal mixing (Eq. 12) retains i) two-rows complex-conjugate and iii) $|U_{e2}| = |U_{\mu 2}| = |U_{\tau 2}| = 1/\sqrt{3}$, dropping ii) $J_{CP} = 0$. Altarelli-Feruglio mixing [12] completes the set, dropping iii) $|U_{e2}| = |U_{\mu 2}| = |U_{\tau 2}| = 1/\sqrt{3}$, but retaining i) two rows complex-conjugate (equal in modulus) and ii) $J_{CP} = 0$. Of course, only tri-bimaximal mixing itself retains all three of the above symmetries (and is furthermore completely defined by them).

There is no implication here that the list of one-parameter generalisations of tri-bimaximal mixing is exhausted. For example, we have also considered CP-conserving and CP-violating analogues of Eq. 9 and Eq. 12 which leave unmodified the first column (rather than the second column) of the tri-bimaximal mixing matrix, altough we judge these somewhat less 'natural' than those presented above.

Clearly, less-constrained ansatze are obtained dropping two symmetries simultaneously, ie. retaining only one. The (three-parameter) set of all mixings with $J_{CP} = 0$

(see Section 3) is obviously too unconstrained to be useful, and the two parameter mixing ansatz, having rows 2 and 3 of the MNS matrix complex-conjugate (equal in modulus) has already been discussed in Section 4. Our final, and perhaps our most useful, two-parameter generalisation of tri-bimaximal mixing drops i) two rows complex conjugate and ii) $J_{CP} = 0$, but retains iii) $|U_{e2}| = |U_{\mu 2}| = |U_{\tau 2}| = 1/\sqrt{3}$. This mixing ansatz is defined (in the circulant basis) simply by a neutrino mass-matrix with four off-diagonal 'texture zeroes' [9] (ie. by a neutrino mass-matrix with effective block-diagonal form, cf. Eq. 3), and it interpolates smoothly between tri-ϕmaximal and tri-χmaximal mixing. Appealing to the unitarity of the MNS matrix, one might even claim that the block-diagonal constraint (ie. the presence of the texture zeroes, Eq. 3) is enough to *explain* the solar data [5] *and* to explain maximal mu-tau mixing at the atmospheric scale [6], given $|U_{e3}|$ small from reactors [7].

We do not know which (if any) of the above symmetries will survive as experiments become more refined. With CP-violation an established feature of quark mixing [15] [16], and CP-violation in the lepton-sector seen universally as a crucial goal experimentally [14], we are tempted to give most emphasis here to our one-parameter CP-violating ansatz 'tri-χmaximal' mixing, Eq. 12, as our most interesting and predictive ansatz. In any case, however, in practical terms, tri-ϕmaximal mixing and tri-χmaximal mixing represent the two extremes that one has necessarily to consider experimentally, and we have also made it clear how best to interpolate between them. Perhaps the most remarkable thing is that tri-bimaximal mixing itself (which adequately represents current experimental observation) comprises so many symmetries.

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